UDC: 534-16
OECD: 01.03.AA

Flexural Vibrations of Non-Uniform Cantilever Beams Observed in Experimental Investigations on the Dynamics of Fleuret, Épée and Sabre

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Abstract

Numerical and experimental investigations of non-uniform beams under static as well as under dynamic loads are required in many engineering applications. However, this topic is also relevant in sports. In particular the mechanical model of a non-uniform cantilever beam could be useful to analyze the dynamical behavior of the blades used for the weapons of the Olympic fencing competitions. These sporting arms are known as fleuret, épée and sabre. They can be compared in weight and length, but differ significantly in the design of the individual cross-section. However, especially excellent athletes such as Alexander Anatoljewitsch Romankow, Philippe Riboud or Anja Fichtel are able to perform fencing on a high level even if they have to use a weapon that differs from the favorite one. For this reason it is interesting to compare basic dynamic properties of these special sport equipment. Therefore, this paper presents novel results that describe the dynamics of fleuret, épée and sabre using the results of simple experiments. It is (only) aimed (i) to highlight basic phenomena, (ii) to compare the results, and (iii) to reflect the findings with the (non-professional) experience of the author in practicing his sport as athlete and coach.

Keywords: structural dynamics, non-uniform beam, experimental investigation, sporting arms.

Изгибные колебания неоднородной консольной балки, наблюдаемые при экспериментальных исследованиях динамики флереты (рапиры), шпаги и сабли

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Аннотация

Численные и экспериментальные исследования неоднородных балок как при статических, так и при динамических нагрузках требуются во многих инженерных применениях. Однако эта тема актуальна и в спорте. В частности, механическая модель неоднородной консольной балки может быть полезна для анализа динамического поведения лезвий, используемых для оружия олимпийских соревнований по фехтованию. Это спортивное оружие известно как флерет, шпага и сабля. Их можно сравнить по весу и длине, но они существенно отличаются конструкцией отдельного поперечного сечения. Однако особенно отличные спортсмены, такие как Александр Анатольевич Романков, Филипп Рибу или Аня Фихтель, способны выполнять фехтование на высоком уровне, даже если им приходится использовать оружие, которое отличается от любимого. По этой причине интересно сравнить основные динамические свойства

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этих специальных спортивных снарядов. Поэтому в данной статье представлены новые результаты, описывающие динамику флереты, шпаги и сабли с использованием результатов простых экспериментов. Это (только) направлено на то, чтобы (i) выделить основные явления, (ii) сравнить результаты и (iii) отразить полученные результаты с (непрофессиональным) опытом автора в занятиях спортом в качестве спортсмена и тренера.

Ключевые слова: *структурная динамика, неоднородная балка, экспериментальное исследование, спортивное оружие.*

Introduction

From the engineering point of view the blades of the sporting arms fleuret, épée and sabre, compare figure 1, can be understood as cantilever beams with non-uniform cross section. The latter changes significantly from the mounting (close to the holder) to the top. All different blades are flexible in horizontal as well as in vertical direction. However, it is not identical. Fleuret and épée are designed to hit the vest of the opponent with the tip of the blade (followed by elastic or in many cases also in-elastic buckling), the sabre is used strike in order to set a touché. Therefore, fleuret and épée are more flexible in the vertical direction compared to the horizontal direction. The opposite is true for the sabre that need to be stiffer in the vertical direction.

Fig. 1. Fleuret (top), épée (middle) and sabre (bottom) analyzed in the experiments

Structural elements showing similar properties are well documented in literature. Basic and advanced principles of structural dynamics have been described in detail by Timoshenko [1] – a relevant starting point for all engineers working in structural dynamics. A simplified analytical model for vibrations of non-uniform flexural beams with viscoelastic properties has been proposed in [2]. Closed-form solutions for axially functionally graded Timoshenko beams having uniform cross-section are discussed in [3]. Isotropic beams with continuously changing cross-section have been studied in [4], providing solutions for clamped-clamped as well as for simply supported boundaries at both ends.

Free vibrations of non-uniform cross-section and axially functionally graded Euler-Bernoulli beams considering various boundary conditions using the differential transformation method to derive the solution have been discussed in [5]. A higher order continuum theory, which contains higher-order equilibrium relation for moments of couple stress in addition to well-known classical equilibrium relations for forces and moments of forces and only one additional material length scale parameter has been applied to analyse the vibrational behaviour of axially functionally graded tapered microbeams using the Rayleigh–Ritz method to obtain numerical solutions [6]. Modal characteristics of a rectangular beam having a variable cross-section with multiple cracks has been discussed in [7] considering different temperatures. The finite element method has been applied in [8] to calculate both natural frequencies and modal shapes of so called multistep nonuniform beams.

The references cited above are focussed on mathematical modelling as well as on numerical evaluation of the derived models. However, it is also possible to find references that combine theoretical and experimental work related to the scope of the present paper. Numerical and experimental investigations of a cantilever beam structure considering nonlinearities in geometry have been reported in [9]. The static deflection of initially curved beams, having a shape that can be compared to the shape of fleuret, épée and sabre, has been discussed in [10]. The dynamical behaviour of a non-uniform beam considering a transversely and axially eccentric tip mass (especially interesting for fleuret and épée) has been analysed in [11].

It turns out that all these excellent reverences cannot directly be used to highlight basic phenomena known from fencing with fleuret, épée and sabre. The theories applied in the theoretical work are sophisticated, closed-form solutions for clamped-free boundary conditions are not easy to derive and numerical tools such as the finite element method have to be applied to solve many of the proposed models. Furthermore, experimental data obtained from vibration measurements that can be used to characterize the dynamics of the sporting arms are still missing in literature. For this reason, the present paper proposes an approach that combines the classical theory of Euler-Bernoulli applied to cantilever beams with novel experimental results characterizing the flexural vibration observed for fleuret, épée and sabre performing simple experiments. Because of the experimental approach, it is not necessary to apply a sophisticated mathematical model. However, the findings presented in this paper are relevant as benchmark for numerical investigations or for the validation of numerical models.

1. Short Comments on Structural Dynamics

In order just to connect the experimental investigations with the principles of structural dynamics, it is sufficient to apply the classical theory of Euler-Bernoulli for a uniform beam. Following this approach it is, if necessary, at least possible to prove that natural frequencies determined in the experiments have been determined in the "corrected" frequency range. This, compared to some of the references, simple approach, is based on the equation of motion, compare equation (1), in which w is the depending variable representing the displacement-field that depends on time and space. The bending stiffness is represented by EI. ρ is the density of the material and A is the cross section.

$$\rho A \cdot \ddot{w} + [EI \cdot w'']'' = 0, \tag{1}$$

To model a cantilever beam it is necessary to formulate proper boundary conditions, compare equation (2), in which L is the length of the beam and t is the time.

$$\begin{aligned} w(0,t) = 0, \qquad w'(0,t) = 0, \\ w''(L,t) = 0, \qquad w'''(L,t) = 0, \end{aligned} \tag{2}$$

Following the principles of structural dynamics, compare [1], the closed-form solution for the natural frequency reads

$$f_{0i} = \frac{(\lambda_i L)^2}{2\pi} \cdot \sqrt{\frac{EI}{\rho A \cdot L^4}}. \tag{3}$$

Table 1 contains the normalized eigenvalues λ, L of the first three eigenmodes. These values have been found considering constant distribution of mass (ρA) and bending stiffness (EI).

Table 1
Normalized eigenvalues of cantilever Beams

	Number of eigenvalue		
	1	**2**	**3**
λ, L	1.8751	4.6941	10.996

Considering the arithmetic mean of the cross section areas listed in table 2 as well as typical values for Young's modulus 1.8e5 N/mm^2 and density 8.0 g/cm^3 (maraging steel), it is possible to calculate estimates for the first three natural frequencies for fleuret, épée and sabre according to vertical deflections. These estimates are given in table 3. They will later on be used to verify the experimental results.

Table 2
Cross sections at different positions (width x height)

W x H	**Position**		
	Mounting	**Middle**	**Top**
Fleuret	7 x 12	3 x 4	3 x 2
Épée	20 x 9	7 x 4	6 x 4
Sabre	7 x 18	3 x 6	2 x 5

Table 3
Numerical estimates for natural frequencies

f_0 / Hz	**Number of natural frequency**		
	1	**2**	**3**
Fleuret	5.0	31.5	88.0
Épée	4.9	30.5	85.4
Sabre	8.4	52.6	147.3

2. Experimental Investigations

In order to determine time-dependent as well as frequency dependent dynamic properties all sport arms have been analyzed. The kind of mounting is shown in figure 2 using the example of the fleuret. A typical piezo-electric sensor mounted on the blade has been used for the measurements. Vertical and horizontal vibrations have been determined subsequently be changing the mounting position with a rotation of 90^o. Impulse input as well as step input have been used to excite the vibrations. To realize the impulse input, the blade has been hit by another blade in the middle of the blade. This can be compared to the situation in the competition. The step input (with a magnitude of several centimeter) has been realized manually by imposing a deflection at the tip of the blade. In both situations it has been assured that non-linear effects have been omitted. All measurements have been recorded using a sampling rate of 44.1 kHz to obtain a high frequency resolution.

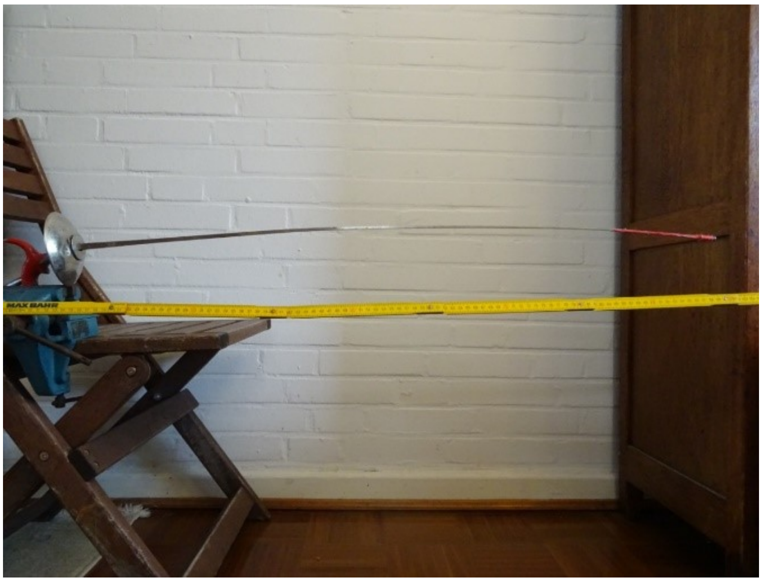


Fig. 2. Mounting of test objects before measurement (shown via the example for the fleuret)

For épeé and sabre the impulse response and the step response is shown in figure 3. It can be found that the impulse response decreases in a very short period of time. Due to the kind of excitation, the step response seems to be dominated by the first bending modes. This can clearly be observed for the sabre, compare figure 3 (right).

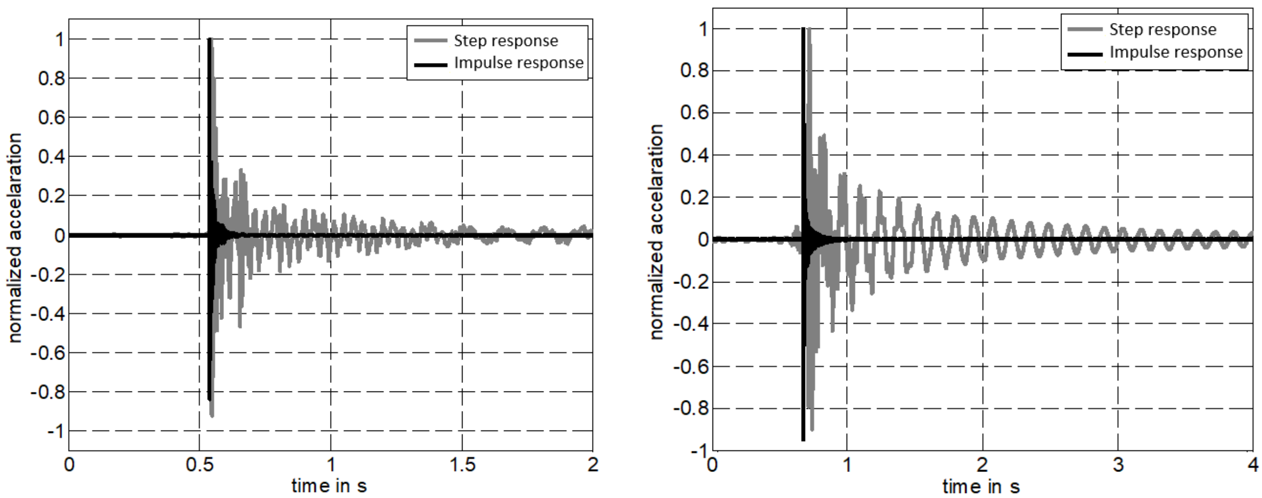


Fig. 3. Free vibration of épée (left) and sabre (right). Impulse response (black line), Step response (gray line)

In order to estimate the decay time T60, the time-dependent behaviour has also been analyzed using a logarithmic sale as shown in figure 4. It has been found that the decay time varies between 0.2 s and 0.3 s. This relatively short time periods are essential for the referee that has to analyze the fight in order to decide who is allowed to set a point because of a successful attack or (alternatively) due to a successful riposte.

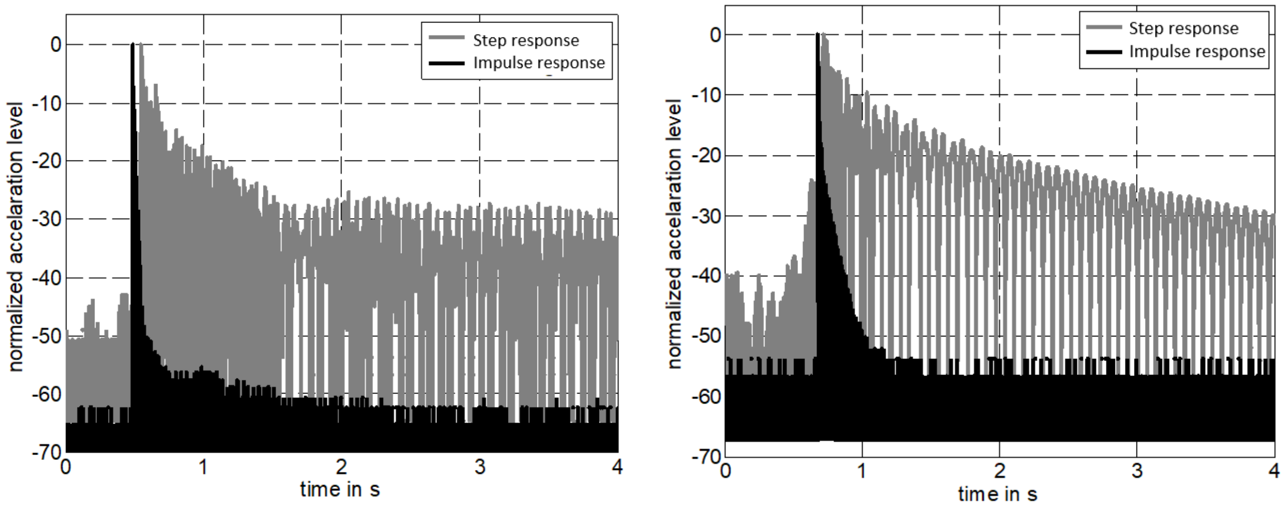
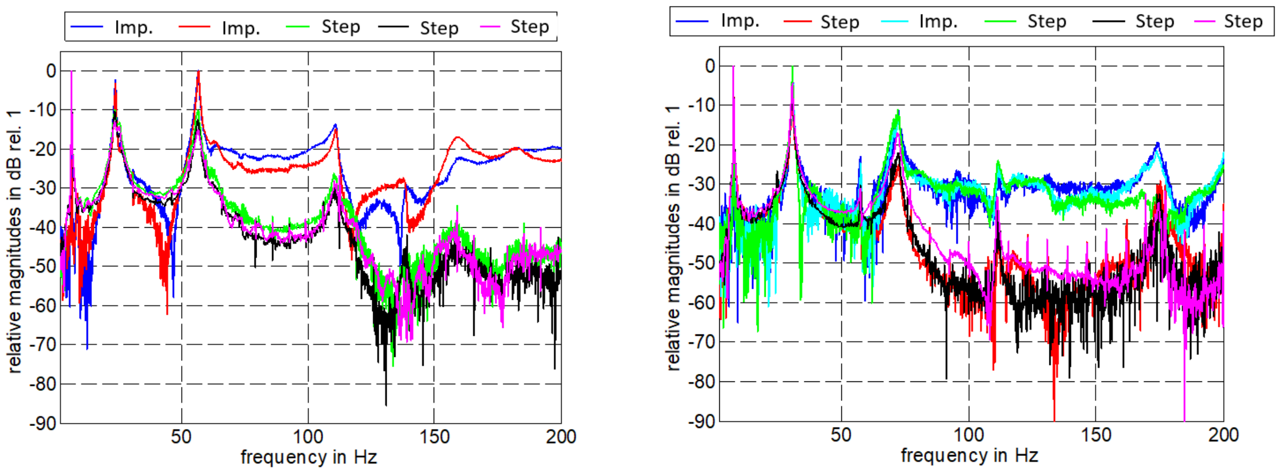


Fig. 4. Decay of acceleration lever for épée (left) and sabre(right). Impulse response (black line), Step response (gray line)

The frequency-dependent behavior is shown in figure 5. It contains the Fourier transforms of several measurements performed for fleuret, épeé and sabre without additional filtering. For this reason the effect of higher frequencies can be detected in all curves. However, the results shown in this figure proof that the basic dynamic phenomena can be reproduced even if such a simple test procedure is used.



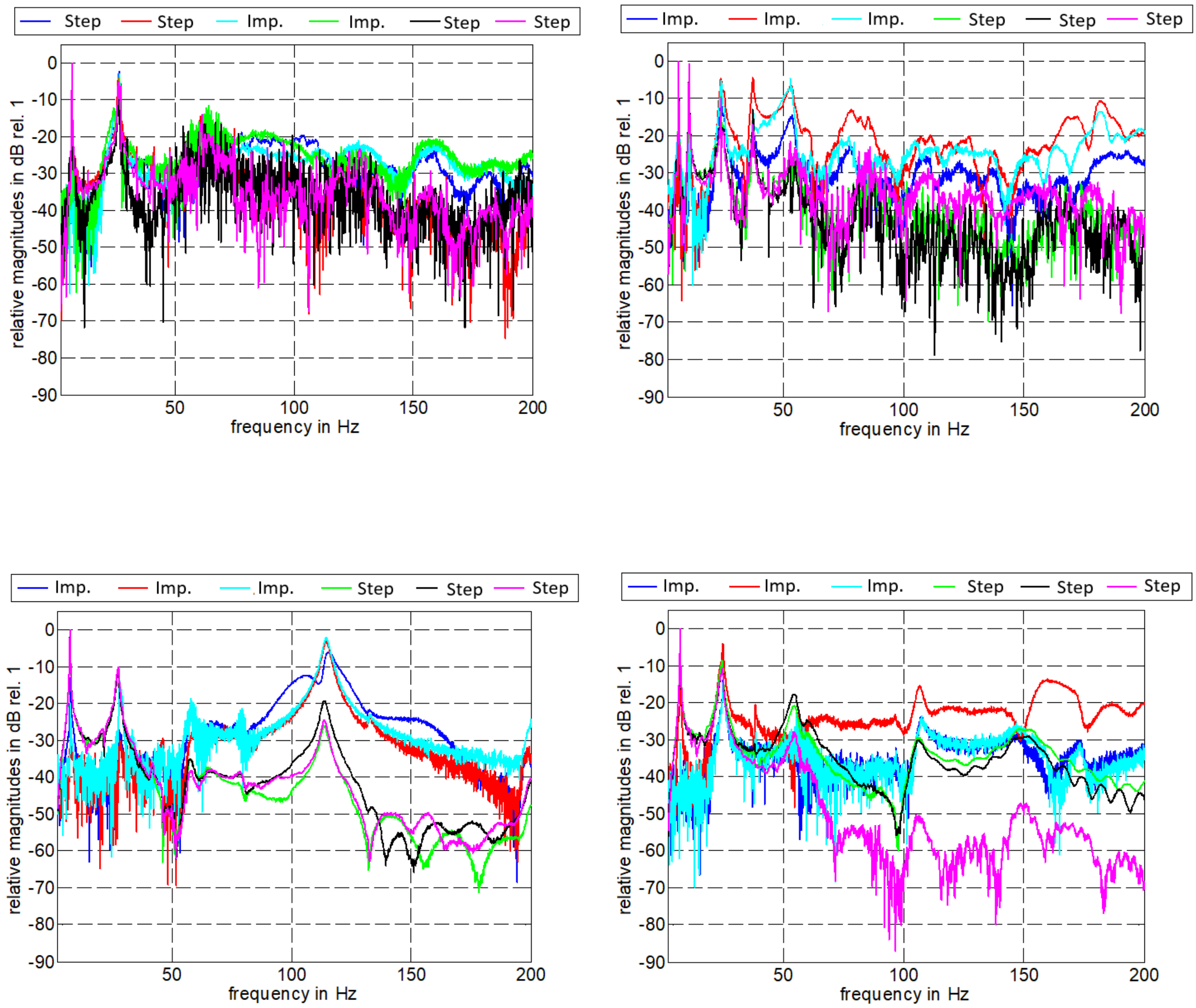


Fig. 5. Magnitude response calculated for impulse input (Imp.) and step input (Step) for fleuret (top), épée (middle) and sabre (bottom). Left column – vibration in vertical direction, right column – vibration in horizontal direction

A modal characteristic can be observed in all measurements below 50 Hz. In this frequency range, the modal damping seems to be low. However, at higher frequencies the half-band-width increases. This might be caused by the effect of sound radiation that is more relevant at the mid and high frequency range.

In order to detect the resonance frequencies as well as to quantify the amount of damping at the first three resonances, linear averages of the curves shown in figure 5 have been used. The resulting curves are shown in figure 6.

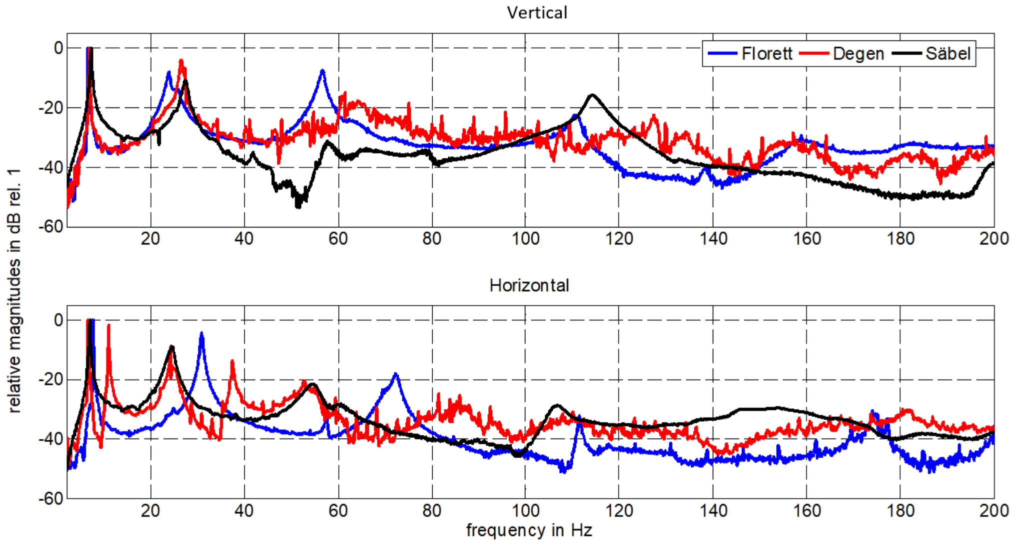


Fig. 6. Frequency response for fleuret (blue curve), épée (red curve) and sabre (black curve) in the lower frequency range

The resonance frequencies obtained for the fleuret from the curves shown in figure 6 are summarized in table 4. Especially the first two resonances are close to the natural frequencies listed in table 3. It is interesting to notice that the first mode occurs for all analyzed sporting arms at 7.0 Hz. This could help to understand why an athlete specialized in fleuret fencing is also capable to perform a Coupé with the épée in order to set a point directly behind the bell guard.

Assuming that the first bending mode dominates the dynamics of all analyzed sporting arms it is also possible to understand why athletes prefer to hit the weapon of the opponent before starting a Coupé. With such a short blade contact (Battuta) the first bending mode is exited and the (dynamic) stiffness of the blade is reduced. For this reason, less energy is required to induce a bending of the blade that makes it possible to attack the back of the opponent that cannot be reached directly. At higher frequencies the difference in the geometry leads to different results for higher resonance frequencies.

Table 4
Resonance frequencies determined in measurements

f_0 / Hz	**Number of natural frequency for fleuret**		
	1	**2**	**3**
Vertical vibration	7.0	24.0	57.0
Horizontal vibration	7.0	31.0	72.0

Using the method of the half-band-with for each individual resonance, the modal damping parameters have been estimates. These results are summarized in table 5 and table 6. They prove that the motion of the fleuret, the épeé and the sabre undergoes only a small amount of damping in the first three modes. This means that the athlete must not waste energy by dissipation or in other words, only a small amount of energy is required to "use" the dynamics of the weapon in competition. This could be an explanation for the fact that excellent athletes are able to perform fencing on a high individual level in the finals at the late afternoon, even if the competition starts with the qualification rounds in the early morning.

Table 5
Modal damping for vertical vibration

D in %	**Mode number**		
	1	**2**	**3**
Fleuret	0.4	1.9	1.0
Épée	0.4	1.4	N.A.
Sabre	1.4	2.0	2.0

Table 6
Modal damping for horizontal vibration

D in %	**Mode number**		
	1	**2**	**3**
Fleuret	0.3	0.8	1.2
Épée	0.4	0.3	2.1
Sabre	1.3	2.8	3.2

Conclusions

The present work presents experimental results that have been performed to study the dynamical behavior of fleuret, épeé and sabre in time domain as well as in frequency domain. The applied method is simple, however the results can be verified with simple analytical analysis. Furthermore, it is easy to repeat the investigations. The main findings can be summarized as follows:

- All analyzed sporting arms are structural elements with a small amount of modal damping. A value of 3.2% has been detected as the maximum damping ratio.
- The first bending mode can be excited for all analyzed weapons at about 7.0 Hz.
- At higher frequencies the different non-uniform geometry causes different behavior for the same sporting arm in horizontal and vertical direction. Furthermore, the difference between the dynamics of fleuret, épée and sabre increases, if higher frequencies have to be taken into account.

The presented results might be interesting for both athletes and coaches, because it is possible to understand experiences known from training and competition. For example it is possible to explain that the performance of a Coupé becomes easier after a short blade contact, because the dynamic stiffness is lowered, if a fleuret or épeé vibrates in a resonance mode.

However, from an engineering point of view, the results could also be interesting. They could be used to validate analytical as well as numerical models used to analyze the dynamics of non-uniform beams. Furthermore, the results could also be of interest for the development of robotics, capable to handle thin flexible structures with a low amount of modal damping.

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